Roll No. | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilators

1. ______________________

2. ______________________

1615

Question Booklet Series **A**

PAPER–II **Question Booklet No.**

Subject Code : 16

OMR Sheet No. ....

(To be filled by the candidate)

PHYSICAL SCIENCES

Time : 1 Hour 15 Minutes *Maximum Marks: 100*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet and the OMR Sheet Number should be entered on this Question Booklet.
3. This paper consists of fifty (50) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

[Please Turn Over]

1. What is the value of $\int_{-\alpha}^{\alpha} dx\, e^{-x}\, \delta'(x)$, where $\delta'(x) = \frac{d}{dx}\left(\delta(x)\right)$?

(A) 0
(B) 1
(C) π
(D) e^{π}

2. If λ_1, λ_2 and λ_3 are the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix},$$

then the value of $\sum_{i=1}^{3} \lambda i^2$ is

(A) 41
(B) 49
(C) 53
(D) 57

3. A function $f(x) = A\, Sin^2 x;\ -\infty \leq x \leq +\infty$ What is its average value?

(A) Zero
(B) $\frac{A}{2}$
(C) $\frac{A}{2\pi}$
(D) $\frac{\pi}{2}A$

4. If $\Phi(\xi) = \oint_c \frac{3z^2+5z+7}{z-\xi} dz$ where the contour C (anticlockwise) is the circle $|z| = 3$, what is the value of $\Phi'(1+i)$?

(A) $7 + 5i$.
(B) $6 + 7i$
(C) $8 + 3i$
(D) $11 + 6i$

5. A force $\vec{F} = (5\hat{\imath} + 2\hat{\jmath})$ Newtons acts at the point $(2, 1, 0)$ metres. What is its moment about the origin?

(A) $-\hat{k}$ N–m
(B) $+\hat{k}$ N–m
(C) $-\hat{\jmath}$ N–m
(D) $9\hat{k}$ N–m

6. A cube has a total surface area S. What is the value of the surface integral

$$\underset{\substack{\text{entire surface}\\ \text{of cube}}}{\iint} \vec{r}.\overrightarrow{ds}$$

(A) Zero
(B) $\frac{S}{6}$
(C) $\left(\frac{S}{6}\right)^{3/2}$
(D) $3\left(\frac{S}{6}\right)^{3/2}$

7. The differential equation

$$x^3 y^{11} - 6y = 0$$

has

(A) a singularity at $x = 1$.
(B) a removable singularity at $x = 0$.
(C) an irremovable singularity at $x = 0$.
(D) no singularity at $x = 0$.

8. For the equation $\frac{dy}{dx} + y = x$, the particular integral is

(A) $(x - 1)$
(B) $(x - 1)e^{-x}$
(C) e^{-x}
(D) $e^{-x}[1 + e^{+x}(x - 1)]$

9. For a particle moving uniformly, the function $F = ax - b\frac{pt}{2m}$ (a and b being two constants and other symbols having usual significance) is

(A) a constant of time provided $a = b$.
(B) a constant of time provided $a = \frac{b}{2}$.
(C) a constant of time provided $a = -b$.
(D) not a constant of time.

[Please Turn Over]

10. A cylinder of mass M and radius R is rolling down without slipping on a stationary inclined plane of angle of inclination θ. The number of generalized co-ordinates required to describe the motion of this system is

(A) 1
(B) 2
(C) 4
(D) 6

11. The Lagrangian of a point particle, dropping dimensionful constants, is given by $L = \dot{x}^2 - x^2$. Which term, added to the Lagrangian will not change the equation of motion?

(A) $\ddot{x}$
(B) $(x + \dot{x})^2$
(C) $x\dot{x}$
(D) $x\dot{x}^2$

12. The Lagrangian of a particle of mass m is given by $L = \frac{m|\vec{v}|^2}{2} + C\vec{r}.\vec{v} - V(r)$, where C is a constant and all other symbols have their usual significance. If $\overrightarrow{P_C}$ is the canonical momentum, then the Hamiltonian is

(A) $H = \frac{1}{2m}(\vec{p_c} + C\vec{r})^2 + V(r)$
(B) $H = \frac{1}{2m}(\vec{p_c} - C\vec{r})^2 + V(r)$
(C) $H = \frac{\vec{p_c}^2}{2m} + V(r)$
(D) $H = \frac{\vec{p_c}^2}{2m} + C^2r^2 + V(r)$

13. A particle of mass m moves on X-axis under potential $V(x) = a + bx + cx^2$ [a, b, c are positive constants]. If the system is displaced slightly from its equilibrium position, then it

(A) performs simple harmonic motion with period $2\pi\sqrt{\frac{m}{2c}}$.
(B) performs simple harmonic motion with period $2\pi\sqrt{\frac{ma}{2b^2}}$.
(C) moves with constant velocity.
(D) moves with constant acceleration.

14. If the generating function for a canonical transformation is $F_2 = \sum_i q_i P_i$

(A) $Q_i = q_i,\ p_i = P_i$
(B) $Q_i = {q_i}^2,\ p_i = {P_i}^2$
(C) $Q_i = q_i,\ p_i = -P_i$
(D) $Q_i = -q_i,\ p_i = P_i$

15. A satellite moving in an elliptical orbit is 400 Km above the earth's surface at its farthest point and 200 Km above at its closest point. If Radius of Earth= $6 \cdot 4 \times 10^3$ Km, the eccentricity in the satellite's orbit is

(A) 0·015
(B) 0·025
(C) 0·030
(D) 0·035

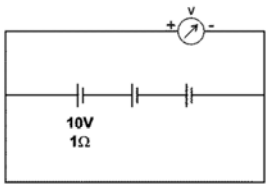
16. A cube at rest has edges of length L_0. If it starts moving parallel to one edge at velocity $\vartheta = \frac{c}{2}$, what will be the fractional change in volume?

(A) Zero
(B) $1 - \frac{\sqrt{3}}{2}$
(C) $\frac{1}{4}$
(D) $1 - \frac{3\sqrt{3}}{8}$

17. The kinetic energy of a particle is equal to its rest energy. Its velocity is

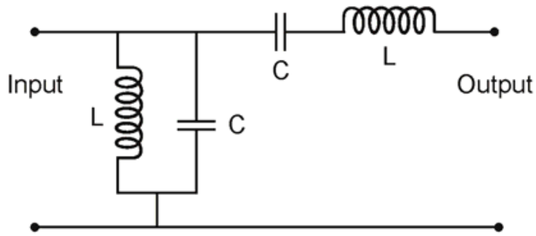
(A) $\frac{c}{2}$
(B) $\frac{c}{\sqrt{2}}$
(C) $\frac{3c}{4}$
(D) $\frac{\sqrt{3}c}{2}$

18. Three identical batteries, each of e.m.f. 10V and internal resistance 1Ω, are connected in series and short-circuited as shown. What is the reading of the ideal voltmeter?



(A) 0
(B) +30V
(C) −30V
(D) $+\frac{30}{\sqrt{2}}$V

19. The circuit



(A) is a low pass filter.

(B) is a high pass filter.

(C) is a band pass filter.

(D) is a band stop filter.

20. In an electromagnetic wave, the electric field $\vec{E} = E_0\hat{k}\ Cos\ \pi\ (0 \cdot 4y - 1000t)$. The magnetic field $\vec{B}$ will be

(A) $(0 \cdot 4i)E_0\ Cos\ \pi\ (0 \cdot 4y - 1000t)$.

(B) $(4 \times 10^{-3})E_0\hat{\imath}\ Sin\ \pi(0 \cdot 4y - 1000t)$.

(C) $(4 \times 10^{-4})E_0\hat{\imath}\ Cos\ \pi\ (0 \cdot 4y - 1000t)$.

(D) $(4 \times 10^{-4})E_0\hat{\jmath}\ Cos\ \pi\ (0 \cdot 4y - 1000t)$.

21. If the electric field associated with a plane wave is given by

$E_x = 20\ Sin\ (1 \cdot 5\pi \times 10^{-7}t - 0 \cdot 1\pi z)\ Vm^{-1}$,

the velocity of electromagnetic wave propagation is

(A) $3 \cdot 0 \times 10^8\ ms^{-1}$

(B) $5 \cdot 0 \times 10^7\ ms^{-1}$

(C) $1 \cdot 0 \times 10^7\ ms^{-1}$

(D) $1 \cdot 5 \times 10^8\ ms^{-1}$

22. Which of the following expressions for a vector potential $\vec{A}$ does not represent a uniform magnetic field of magnitude B_0 along the z-direction?

(A) $\vec{A} = (0, B_0x, 0)$

(B) $\vec{A} = (-B_0y, 0, 0)$

(C) $\vec{A} = (B_0\, {}^{x}/_{2}, B_0\, {}^{y}/_{2}, 0)$

(D) $\vec{A} = \left(-B_0\, {}^{y}/_{2}, B_0\, {}^{x}/_{2}, 0\right)$

23. Maxwell's Equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{dt}$$

is the mathematical statement of which law?

(A) Gauss'

(B) Faraday's

(C) Ampere's

(D) Biot-Savarts'

24. A point charge e is flowing along the x-direction with a velocity v and passes through the origin at $t = 0$. The one-dimensional current density at any time t is

(A) ev

(B) $e\delta(x - vt)$

(C) $ev\ \delta(x - vt)$

(D) $ev\ \delta(x)$

25. The sum of two oppositely rotating circularly polarized waves of equal amplitude will be

(A) a linearly polarized wave.

(B) a circularly polarized wave.

(C) an elliptically polarized wave.

(D) an unpolarized wave.

26. At the boundary of two dielectric media, the electric field lines are refracted according to the relation

(A) $\epsilon_1\ sin\ \theta_1 = \epsilon_2\ sin\ \theta_2$

(B) $\epsilon_1/sin\ \theta_1 = \epsilon_2/sin\ \theta_2$

(C) $\epsilon_1\ tan\ \theta_1 = \epsilon_2\ tan\ \theta_2$

(D) $\epsilon_1 cot\ \theta_1 = \epsilon_2\ cot\ \theta_2$

θ_1 and θ_2 are the angles of incidence and refraction of the respective media with relative permittivities $\epsilon_1$and ϵ_2.

27. Two containers are at same temperature, one contains hydrogen and the other oxygen. The ratio of average speed of hydrogen and oxygen molecules is

(A) 1 : 1
(B) 1 : 4
(C) 4 : 1
(D) 16 : 1

28. An ideal gas expands isothermally from V to $2V$. The work done is

(A) $RT\ ln\ 2$
(B) $2RT\ ln\ 2$
(C) RT
(D) 0

29. The peak of Maxwell-Boltzmann distribution for velocity shifts towards a higher value of velocity

(A) at higher temperature along with decrease in height.
(B) at higher temperature along with increase in height.
(C) at lower temperature along with decrease in height.
(D) at lower temperature along with increase in height.

30. For a Bose gas, which of the following statements about its chemical potential μ is necessarily correct?

(A) $\mu = 0$
(B) $\mu \leq 0$
(C) $\mu \leq \epsilon_0$, where ϵ_0 is the minimum energy of the system.
(D) There is no restriction on μ.

31. At the transition temperature of a first order phase transition for a system,

(A) the specific heat diverges and the entropy remains the same.
(B) the specific heat diverges and the entropy has finite discontinuity.
(C) the specific heat remains unchanged and the entropy has finite discontinuity.
(D) the specific heat has finite discontinuity and the entropy diverges.

32. An ideal gas is expanding such that pT^2= constant. The coefficient of volume expansion $\left(\frac{dV}{dT}\right)$ of the gas is

(A) $\frac{3}{T}$
(B) $\frac{3V}{T}$
(C) $\frac{2}{VT}$
(D) $\frac{V}{T}$

33. The phase trajectory of a particle moving according to the relation $x = ut + \frac{1}{2}ft^2$ (symbols having their usual significance) is

(A) Straight line
(B) Parabola
(C) Ellipse
(D) Hyperbola

34. Temperature at which the heat capacity of gaseous hydrogen molecules changes from $\frac{3}{2}k\ to\ \frac{5}{2}k$ is given by the expression (assuming all the odd rotational levels are missing)

(A) $30\frac{Bch}{k}$
(B) $12\frac{Bch}{k}$
(C) $2\frac{Bch}{k}$
(D) $6\frac{Bch}{k}$

35. Consider a system of eight non-interacting, identical particles of spin 3/2 and mass m in a one dimensional box of length L. The minimum excitation energy in the unit of $\frac{\pi^2\hbar^2}{2mL^2}$ is

(A) 3
(B) 5
(C) 7
(D) 9

36. In the S_z diagonal representation a spin $\frac{1}{2}$ particle is in the state $\binom{1}{2}$. If S_x is measured in this state, the probability of obtaining the value $+\frac{\hbar}{2}$ is

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{7}{10}$

(D) $\frac{9}{10}$

37. If a, a^+ are creation and annihilation operators for simple harmonic oscillator states, then the operator $(a\, a^+ a^+ a\, a^+ a)$ has eigenvalues

(A) $n^2(n+1)$

(B) n^3

(C) $n(n-1)$

(D) Zero

38. If the Hamiltonian of a system is given by $H = \gamma \hat{\vec{L}}.\hat{\vec{S}}$ then the expectation value of energy in $3P_1$ state is

(A) 0

(B) $\gamma\hbar^2$

(C) $-\gamma\hbar^2$

(D) $\frac{-3\gamma}{4}\hbar^2$

39. If the Clebsch Gordan coefficient relating the angular momentum states$|j_1, m_1, j_2, m_2\rangle$ with $|j_1, j_2, j, m\rangle$ state is written as $\begin{bmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{bmatrix}$ then the values of $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 1 & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ and $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ -1 & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ are

(A) 1 and 0

(B) 1 and –1

(C) 0 and 1

(D) 1 and 1

40. The expectation value of the potential energy in the n-th energy eigenstate of a three dimensional harmonic oscillator is given by

(A) $\left(n+\frac{3}{2}\right)\hbar w$

(B) $\left(n+\frac{3}{2}\right)\frac{\hbar w}{2}$

(C) $n\frac{\hbar w}{2}$

(D) $\left(n+\frac{3}{4}\right)\hbar w$

41. A one dimensional simple harmonic oscillator has $H = \frac{p^2}{2m} + \frac{1}{2}k\,x^2$. It is perturbed by $H' = \alpha x^2$, $\alpha > 0$ and $\alpha \ll k$. The energy level shift for the level $E_n = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}}$ will have magnitude

(A) Zero

(B) $\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{\alpha}{m}}$

(C) $\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{\alpha}{k}}$

(D) $\left(n+\frac{1}{2}\right)\hbar\frac{\alpha}{\sqrt{km}}$

42. If $\vec{r}$ and $\vec{p}$ respectively denote the position and linear momentum operator for a quantum particle, then $\vec{r}.\vec{p} - \vec{p}\,.\vec{r}$ has the value

(A) $i\hbar$

(B) $2\,i\hbar$

(C) $3\,i\hbar$

(D) $-2\,i\hbar$

43. Two electrons are in the 1s level of Helium. What is/are its total angular momentum?

(A) 0

(B) 1

(C) 0 and 1

(D) 0,1 and 2

44. $|1\rangle$, $|2\rangle$ *and* $|3\rangle$ form an orthonormal basis of the state space of a quantum mechanical system. If the system is in state $|\Psi\rangle = \frac{1}{2}\big|1\rangle - \frac{1}{3}\big|2\rangle + c|3\rangle$ [c is a constant], the probability that the system is *not* in state $|1\rangle$ is

(A) $\frac{3}{4}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3} + |c|^2$

(D) $\frac{1}{2}$

45. A charged particle is moving in a Coulomb potential. Which one of the following operators will commute with the Hamiltonian?

(A) Kinetic energy

(B) Linear momentum

(C) Angular momentum

(D) Potential energy

46. In order to compare the electronic absorption spectra of three iron (II) complexes in a single plot, you should plot

(A) absorbance against wave length.

(B) absorbance against wavenumber.

(C) extinction co-efficient against concentration.

(D) extinction co-efficient against wavelength.

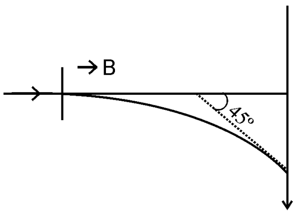
47. A slab of uniform magnetic field deflects a moving charged particle by 45° as shown in the figure below. The kinetic energy of the charged particle at the entry and exit points in the magnetic fields will change in the ratio of

(A) $1 : \sqrt{2}$

(B) $\sqrt{2} : 1$

(C) $1 : 2$

(D) $1 : 1$



48. A sound absorber attenuates the sound level by 20dB. The intensity decreases by a factor of

(A) 10

(B) 100

(C) 1000

(D) 10000

49. Which one of the following functions, *y= f(x)*, can not be fitted with experimental data using linear least square fit method even after variable transformation?

(A) $y = a_o + a_1 x + a_2 x^2$

(B) $y = a_o + \frac{a_1}{x^2}$

(C) $y = \sin(a_o x)$

(D) $y = x^{a_1}$

50. The reversal time of electric field between the dees of a cyclotron to accelerate protons is 1ns. Magnetic flux density employed in it will be (mass of the proton is taken as $1{\cdot}6 \times 10^{-27} kg.$)

(A) $1{\cdot}4$ weber. m^{-2}

(B) $2{\cdot}8$ weber. m^{-2}

(C) $3{\cdot}14$ weber. m^{-2}

(D) $5{\cdot}6$ weber. m^{-2}

Rough Work

Rough Work

Rough Work

Rough Work